Development of Well-Structured Variable Rate Low Density Parity Check Codes for DVB-S2 applications

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ABSTRACT

Both constant and variable Forward Error Correcting (FEC) coding is provided by the DVB-S2 system. Variable FEC can be used for different levels of error protection according to different applications. DVB-S2 system [15] is intended to provide interactive services, which are mainly point-to-point or point-to multipoint, where variable FEC may be combined with the use of return channels, to achieve Adaptive Coding. The novel idea explained in this paper, introduces a well-structured Low Density Parity Check (LPDC) codes, which enables the system to switch between LDPC codes of different rates (9/10, 8/9, 5/6, 4/5...) derived from a base parity check matrix. We introduce a method to achieve variable rate codes starting from higher rate to lower rate codes based on matrix expansion.

1. INTRODUCTION

Variable FEC is essential in the design of practical error control systems. Lots of study and design on variable rate error correcting codes using BCH codes [5], convolutional codes [2,8] and turbo codes [12,13] have been carried out. After recent rediscovery of LDPC codes by MacKay and Neal [10], in contrast to many other codes such as BCH and convolutional codes, LDPC codes offer better performance and even in comparison with Turbo codes they offer lower decoding complexity and have become strong competitors to turbo codes [1]. As demands for using LDPC codes is increasing rapidly within the industry, some authors have proposed methods to design variable rate LDPC codes [6]. These types of LDPC codes are mainly referred to as rate-compatible LDPC codes [3]. In [3], research has been carried out to find puncturing distributions for rate compatible LDPC codes to be able to optimize the performance over a range of rates. At higher rate codes the problem with puncturing appears when the soft-decision decoder fails due to a large percentage of erasures.

In this paper we take a different approach and introduce a method to construct variable rate codes starting from higher rate codes going to lower rate codes based on matrix expansion with no puncturing. The proposed construction method is presented in Section 2 also the structure of their bipartite graph [14] is explained. Simulation results with parameters compliant to DVB-S2 standard are shown in Section 3.

2. Construction Method

In this paper we have considered to use a base parity check matrix, \( H_b \), which can be derived from any type of construction. Then to extend this matrix to a set of larger matrices supporting different code rates, each element of \( H_b \) is replaced by another matrix and its cyclic shifts, which is small in comparison with the size of the base matrix. This method is referred as replacement and expansion.

2.1. Notations and Preliminaries

The \( H_b \) matrix is considered to be regular and full rank of dimension \( M \times N \). A regular LDPC matrix [4], is defined as a binary matrix having exactly \( W_c \) number of ones in each column (column weight) and exactly \( W_r \) number of ones in each row (row weight), where \( W_c < W_r \) and both are small compared to the dimensions of the matrix. With \( H_b \) being a full rank matrix, the code rate can be specified as \( 1-M/N \) or \( 1-W_c/W_r \) [4]. We have constructed \( H_b \) based on the Mackay-Neal LDPC code construction [9,11], however it is also possible to use other construction methods.

Definition 1: Elements of a binary matrix, \( B_{M\times N} \), are represented by \( e(i,j) \in B_{M\times N} \) where \( 1 \leq i \leq M \) and \( 1 \leq j \leq N \) and \( e(i,j) \) is a binary value referring to the element in the \( i^{th} \) row and \( j^{th} \) column of the \( B_{M\times N} \) matrix.

Consider a \( q \times q \) identity matrix, \( I_q \), by inserting number of zero rows evenly among the rows of the \( I_q \), another matrix, \( \alpha_{p \times q} \), with column weight of ‘1’ is constructed where \( p \geq q \) and \( p-q \) is the number of inserted zero rows. As an example, for \( p=9 \) and \( q=6 \), \( \alpha_{9 \times 6} \), is represented in Fig. 1 (a).
Definition 2: Considering a matrix \(\alpha_{pq}\) and \(s\) a natural number, matrix \(\alpha_{pq}^{s}\) is defined as “\(s\)-shifted” \(\alpha_{pq}\), such that:

\[
\forall \ e(i,j) \in \alpha_{pq} \ , \ \forall \ e'(i,j) \in \alpha_{pq}^{s} \quad \text{where} \ 1 \leq i \leq p \ \text{and} \ 1 \leq j \leq q \ , \ \text{we have:}
\]

\[
e((i + s - 1) \mod m, j) = e'(i, j)
\]

As an example, in Fig. 1 (b), “1” shifted \(\alpha_{9,6}\), which is denoted by \(\alpha_{9,6}^{2}\), is represented.

\[
\begin{align*}
\alpha_{9,6} &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \\
\alpha_{9,6}^{2} &= \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

Fig. 1. Matrices \(\alpha_{9,6}\) and \(\alpha_{9,6}^{2}\).

2.2 Matrix Expansion

The concept of matrix expansion is introduced where each element of \(H_b\) is replaced by set of other matrices, all having the dimensions \(p\times q\). In this case considering \(H_b\) expanded matrix to be full rank, the rate can be calculated as:

\[
\text{Rate} = 1 - \frac{(p \cdot M)}{(q \cdot N)}
\]

The replacement and expansion process is carried out row-wise on \(H_b\) such that for each row, traversing from column \(j=1\) to \(N\), the \(j^{th}\) ‘1’ encountered is replaced with the matrix \(\alpha^{\gamma \mod p}\) and each of the zeros of the \(H_b\) with a null matrix of the same dimension \(p\times q\). The resultant matrix is denoted by \(H(H_b, p, q)\).

In order to design a set of codes with fixed code length, \(L\), and different code rates, first we choose parameters \(q\) and \(N\) such that \(L=qN\). From equation (2), we observe that if \(p=q\) then the code rate of the expanded matrix is same as that of the base matrix \(H_b\) and hence for a fixed \(N\) the rate depends only on \(M\). With \(p=q\), the parameter \(M\) can be chosen for the highest desired code rate and then by increasing \(p\), lower code rates can be achieved.

As \(H_b\) does not have any cycles of length 4 (i.e. number of ones in common between and two columns of the matrix is no greater than 1), hence the expanded matrix also has no cycles of length 4.

For \(p>q\), a number of zero rows equal to the \(p-q\) within \(\alpha_{pq}\) and its cyclic shifts \(\alpha_{pq}\) are placed. The idea behind placing zero rows is to increase the ratio between number of rows and the number of columns of the expanded matrix, \(H(H_b, p, q)\), compared to this ratio for \(H_b\) \((M/N)\) to provide lower rate codes as shown in the equation (2). The same time the overall row weight of \(H(H_b, p, q)\) is reduced with the inverse factor \((\sim q/p)\) as shown in Table 1. This results in an increase in the number of parity check constraints resulting in efficient lower rate codes.

2.3 The Structure of the Tanner Graph

Tanner [14] introduced bipartite or Tanner graphs to describe families of codes, which are generalizations of the LDPC codes. It is very convenient to construct message-passing algorithms based on the Tanner graphs. In this section with an example we represent a method in which bipartite graph of \(H(H_b, p, q)\) can be derived from the graph of \(H_b\). The \(H_b\) with dimension \(M \times N\) defines a code in which \(N\) elements of each codeword satisfy a set
of $M$ parity-check constraints. The Tanner graph contains $N$ "bit nodes", one for each bit, and $M$ "check nodes", one for each parity check equation or constraint. Specifically, a branch connects check node $i$ to bit node $j$ if and only if the $i^{th}$ parity check involves the $j^{th}$ bit, or more succinctly, if and only if for $e(i,j) \in H_{M \times N}, e(i,j)=1$ where $1 \leq i \leq M$ and $1 \leq j \leq N$.

The graph of $H(H_b, p, q)$ contains $q.N$ bit nodes, one for each bit, and $p.M$ check nodes, one for each parity check equation. As we replace ones of the $H_b$ by $\alpha$ matrix or its cyclic shifts, the bipartite graph of $H(H_b, p, q)$ follows the same structure as bipartite graph of $H_b$. To represent this graph, using graph of $H_b$, each branch of $H_b$ is replaced with a graph representation of $\alpha$ matrix or its cyclic shifts. This is shown in Fig. 3.

![Graphs](image)

**Fig. 3.** (a) The $i^{th}$ check node and corresponding bit nodes graph of $H_b$; (b) Graph representation of $H(H_b, p, q)$ based on $H_b$; (c) Graph representation of $\alpha_{9 \times 6}$; (d) Graph representation of $\alpha_{9 \times 6}^2$.

### 3. SIMULATION RESULTS

The simulation is performed for code-rates compliant to the DVB-S2 standard [15]. The Code-length is fixed to $L=64800$ bits in accordance with the standard. Simulation is conducted for the 8 different code rates using two different $H_b$ matrices. The first matrix, $H_{1b}$, has dimension of $1350 \times 5400$ and the second matrix, $H_{2b}$, of dimension $540 \times 5400$. The column weight $W_c=3$ is constant for both matrices, whereas the row weight for $H_{1b}$ and $H_{2b}$ varies between 6 to 12 and 15 to 30 respectively from code rates between 1/2 and 9/10. The values of $p$ and $q,$ are also given in the table 1. These codes have been simulated over A White Gaussian Noise and the modulation scheme is chosen to be binary phase shift keying (BPSK). Decoding structure used is sum-product algorithm [4] with maximum number of iterations being limited to 50. The results for code rate 9/10, 8/9, 5/6 and 4/5 are shown in Fig. 4.

<table>
<thead>
<tr>
<th>Code Rates</th>
<th>$W_r$</th>
<th>$W_c$</th>
<th>$p$</th>
<th>$q$</th>
<th>$H_{1b}$ &amp; $H_{2b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>6</td>
<td>3</td>
<td>24</td>
<td>12</td>
<td>$1350 \times 5400$</td>
</tr>
<tr>
<td>3/5</td>
<td>8</td>
<td>3</td>
<td>19</td>
<td>12</td>
<td>$1350 \times 5400$</td>
</tr>
<tr>
<td>2/3</td>
<td>9</td>
<td>3</td>
<td>16</td>
<td>12</td>
<td>$1350 \times 5400$</td>
</tr>
<tr>
<td>3/4</td>
<td>12</td>
<td>3</td>
<td>12</td>
<td>12</td>
<td>$1350 \times 5400$</td>
</tr>
<tr>
<td>4/5</td>
<td>15</td>
<td>3</td>
<td>24</td>
<td>12</td>
<td>$540 \times 5400$</td>
</tr>
<tr>
<td>5/6</td>
<td>18</td>
<td>3</td>
<td>20</td>
<td>12</td>
<td>$540 \times 5400$</td>
</tr>
<tr>
<td>8/9</td>
<td>27</td>
<td>3</td>
<td>13</td>
<td>12</td>
<td>$540 \times 5400$</td>
</tr>
<tr>
<td>9/10</td>
<td>30</td>
<td>3</td>
<td>12</td>
<td>12</td>
<td>$540 \times 5400$</td>
</tr>
</tbody>
</table>

![Table 1](image)

**Fig 4.** Performances for code rates 9/10, 8/9, 5/6 and 4/5 using BPSK modulation.
CONCLUSIONS

A new method performing variable rate LDPC codes has been introduced. The structure of bipartite graph suggests a simple decoding algorithm using graph representation of the base matrix can be devised and the results confirm the effectiveness of this method.

4. REFERENCES


